Time: 20 minutes

1. Note: If S is a set of points, then S' is the set of points **not** in S. On the answer sheet, draw a diagram of three mutually intersecting circles. On the answer sheet, shade in only the region corresponding to the expression $[(A \cap B) \cup (A \cap C) \cup (B \cap C)] \cap [(A \cap B \cap C)']$

2. How many 5-digit numbers have all the following properties?
      a) All 5 digits are different
      b) The first digit is from 2 through 6 inclusive
      c) The last digit is from 3 through 7 inclusive
      d) The middle digit is odd

3. Let S be the set of lattice points P(x, y), where x and y are integers from 0 through 3 inclusive. We call this set $\{P_1, P_2, P_3, P_4\}$ a SQUARESET if P_1, P_2, P_3, P_4 are the vertices of a square (order is not important). How many distinct SQUARESETS are there which are subsets of S? Since order is not important $\{P_1, P_2, P_3, P_4\}$, for example, is not considered different from $\{P_2, P_1, P_3, P_4\}$.

4. We have a two-player game which always has a winner. When A and B play, A beats B 2/3 of the time; when B and C play B beats C 2/3 of the time; and when C and A play, C beats A 2/3 of the time.
      a) If A plays B, then the winner plays C, the probability that A will be the final winner is r
      b) If A plays C, then the winner plays B, the probability that A will be the final winner is s
      c) If B plays C, then the winner plays A, the probability that A will be the final winner is t

Compute the ordered triple (r, s, t)

5. Find the smallest 3-digit number such that both of the following are true:
      a) The number formed by any two of its digits (in either order) is a prime
      b) The number formed by its three digits, (in any order), is a prime

Time: 20 minutes

1. Let a and b be the lengths of the legs of a right triangle with the following properties
    a) All three sides of the triangle are integers
    b) The perimeter of the triangle is numerically equal to the area of the triangle

Compute all possible ordered pairs (a, b) where a<b

2. Consider a diagram that shows a circle in which we have inscribed a rectangle surmounted by a semicircle, thus the outer circle passes through two vertices of the rectangle and is tangent to the semicircle. If the length of the rectangle is 30 and its height is 10, compute the radius of the outer circle.

3. The "centroid" of a triangle is the intersection of its medians. Scalene triangle ABC is reflected through its own centroid, G, the image being triangle $A'B'C'$. If AB = 2BC and the area of triangle $A'B'C'$ is 72, compute the area of the hexagonal region common to both triangle ABC and triangle $A'B'C'$

4. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, compute the ordered pair (x_1, x_2).

5. The graphs of $x^2 + y^2 + 6x - 24y + 72 = 0$ and $x^2 - y^2 + 6x + 16y - 46 = 0$ intersect at four points. Compute the <u>sum</u> of the distances from these four points to the point (-3, 2).