

## Solutions for Meet 4

### Individual Questions

1. If the three angles are  $3x$ ,  $4x$ , and  $5x$ , then  $3x + 4x + 5x = 180$ , and  $x = 15$ . The required

ratio is  $\frac{\sin 45}{\sin 60} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

2. Suppose runners meet at point R. Let  $PQ = d$ ,  $QR = a$ , then if B increases his speed by  $x$  mph,

we have:  $\frac{d-a}{9} = \frac{d+a}{12}$  and  $\frac{d+a}{9+x} = \frac{d-a}{12}$ . From the first equation we find that  $\frac{d+a}{d-a} = \frac{4}{3}$ ,

and from the second we find that  $\frac{d+a}{d-a} = \frac{9+x}{12}$ . So  $\frac{9+x}{12} = \frac{4}{3}$  and  $x = 7$ .

2.  $f(x) = Q(x) \cdot (x-a) + R = \frac{1}{3}Q(x) \cdot 3(x-a) + R$ . The remainder is not affected. Therefore  $(g, h) = (1/3, 1)$

4. Draw a cube and list the given information. If A is at  $(0,0,0)$  and G is at  $(1,1,1)$ , plane BCD has the equation  $x + y + z = 1$ . Line AG has equation  $t < 1,1,1 >$ . H is on both. So  $t + t + t = 1$ ,  $t = 1/3$ . H is  $(1/3, 1/3, 1/3)$ .  $AH = \sqrt{3 * (1/3)^2}$ .  $HG = \sqrt{3 * (2/3)^2}$ . So AH : HG is 1:2.

5.  $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ , so  $(x+y)^2 = xy$  and  $x^2 + xy + y^2 = 0$ . Thus  $\left(\frac{x}{y}\right)^2 + \frac{x}{y} + 1 = 0$ , and

$$\frac{x}{y} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

6. Draw and label a diagram of triangle ABC. Let AM be the median, with M on BC. Create D so that M is the midpoint of AD. Then ACDB is a parallelogram, so  $AD^2 + BC^2 = 2AB^2 + 2AC^2$ . If  $BC = x$ , then  $x^2 + 280 = 298$ , and  $x^2 = 18$ , so  $x = 3\sqrt{2}$ .