

## MONTGOMERY COUNTY MATHEMATICS LEAGUE

Individual for Contest # 2

(No Calculators)

2014-2015

Time: 10 minutes

1. The formula that converts Fahrenheit to Celsius is  $C = \frac{5}{9}(F - 32)$ . For an approximate conversion, one can use  $C = \frac{1}{2}(F - 30)$ . Find the only Fahrenheit reading which is converted to the correct Celsius reading by this approximation.

2. Find the first integer, of 100 consecutive ODD integers whose sum is  $100^{100}$ .

Time: 10 minutes

3. In  $\triangle ABC$ , altitude  $\overline{CD}$  and angle bisector  $\overline{CE}$  are drawn to side  $\overline{AB}$ . If AD = 4, AE = 6, and DB = 9, find EB.

4. Write, in the form  $x^2 + px + q = 0$ , the equation whose roots are those of  $x^2 + 2x + 2 = 0$ , each increased by the reciprocal of the other.



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 Team for Contest # 2
 (Calculators NOT Permitted)
 2014-2015

1. The only road in Circleland is circular, and all three inhabitants of Circleland live on this road. The doctor lives at (4, 14), the lawyer lives at (-1, 9), and the Indian Chief lives at (8, 12). One day, the Indian chief visited the doctor. Being dissatisfied with the results of his visit, he headed directly from the doctor to the lawyer, traveling along the road. Find the minimum length, in units, of his trip from the doctor to the lawyer.

2. Find all ordered triples of numbers (x, y, z) which satisfy the equations xy + x + y = 11, xz + x + z = 17, and yz + y + z = 23

3. The positive root of the equation  $4x^2 + ax + b = 0$  is  $sin18^\circ$ . Find the ordered pair of integers (a, b).

4. Some people agree to share equally in the cost of buying a boat. If ten of them later decide not to buy in, each of those remaining would have to chip in \$1 more. If the sole payment actually occurs after an additional fifteen people drop out, each of those ultimately remaining have to pay \$2 more than (s)he would have had to pay had only the first ten dropped out. How many people originally agreed to share in the cost of buying the boat?

5. In the same plane as square ABCD, the circle with center A and radius  $\overline{AB}$  intersects the circle with  $\overline{BC}$  as diameter at points B and E. Let Q be the midpoint of BE and O be the midpoint of BC. If the area of the square is 20, find the area of  $\Delta ABE$ .