

Solutions for Meet 2

Individual Questions

1. We want $\frac{5}{9}(F - 32) = \frac{1}{2}(F - 30)$, solving $10F - 320 = 9F - 270$, so $F = 50$

2. We have $(x) + (x + 2) + (x + 4) + \dots + (x + 198) = 100^{100}$.
This gives $100x + 99(100) = 100^{100}$, so now $x + 99 = 100^{99}$.
Our answer then becomes $x = 100^{99} - 99$

3. Much of the geometric data is unimportant. We have $DE = AE - AD = 6 - 4 = 2$
Then $EB = DB - DE = 9 - 2 = 7$

4. Let the roots of $x^2 + 2x + 2 = 0$ be a and b . Then $ab = 2$ and $a + b = -2$, using the theorem on sum and product of the roots of a quadratic equation. The required equation is given by $x^2 - \left(a + \frac{1}{b} + b + \frac{1}{a}\right)x + \left(a + \frac{1}{b}\right)\left(b + \frac{1}{a}\right) = 0$, or we can write $x^2 - \left(\frac{(ab+1)(a+b)}{ab}\right)x + \left(ab + \frac{1}{ab} + 2\right) = 0$. Now using the values we have the required equation is $x^2 + 3x + \frac{9}{2} = 0$

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Team Questions

1. Let L, D, and I denote the doctor's, lawyer's, and Indian Chief's homes, respectively. The equation of the perpendicular bisector of \overline{LD} is: $y = -x + 13$, while the equation of the perpendicular bisector of \overline{DI} is: $y = 2x + 1$. Solving, the center of the circle is at (4, 9) and arc LD is one quarter of a circle whose radius has length 5. Thus the required distance is $\frac{5\pi}{2}$.

2. Adding 1 to both sides of each equation we get: $(x + 1)(y + 1) = 12$, $(x + 1)(z + 1) = 18$, $(y + 1)(z + 1) = 24$. Multiplying together, $(x + 1)^2(y + 1)^2(z + 1)^2 = 12^2 \cdot 6^2$, from which $(x + 1)(y + 1)(z + 1) = \pm 72$. Dividing this result, in turn, by the originally derived equations, the TWO possible triples are (2, 3, 5) and (-4, -5, -7)

3. Let $y = 18^\circ$. This gives $\cos 3y = \sin 2y$. Using the identity $\cos 3y = 4\cos^3 y - 3\cos y$, we have $4\cos^3 y - 3\cos y = 2\sin y \cos y$. Since $\cos y \neq 0$, we have $4\cos^2 y - 3 = 2\sin y$, or $4\sin^2 y + 2\sin y - 1 = 0$. Since $y = 18^\circ$ and $\sin y$ is the positive root of $4x^2 + 2x - 1 = 0$, it follows that (a, b) = (2, -1)

4. Let $x =$ #people originally, and let each original share be $\$y$. Then the cost of the boat was $\$xy$. The equations arising from the problem are: $(x - 10)(y + 1) = xy$, and $(x - 25)(y + 3) = xy$. When we simplify we get $x = 100$

5. Draw a diagram, and label with other letters, and notice $(BQ)(AO) = (AB)(BO)$. Since $AO = 5$, $BQ = 2$, and $AQ = 4$. The area = 8