

MBMT Sprint Round — Euclid

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **25** questions. You will have **30** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

- _____ 1. What is the remainder when the positive integers from 1 to 7 are added together and then divided by 6?

- _____ 2. Tuning Drum and Celery Hilton are campaigning to become the president of the United States. Drum wishes to build a wall between the US and Mexico while Hilton wishes to tear down the wall Drum builds. If the border between the US and Mexico is 2000 miles and Drum can build at a rate of 5 miles per day while Hilton can tear down the wall at a rate of 1 mile per day, how many days will it take for the wall to be completed?

- _____ 3. What is the units digit of the product of the first 10 primes?

- _____ 4. 1 bottle of an energy drink contains 2000 mg of caffeine. 4.4 bottles will disable someone weighing 140 pounds. If the number of bottles required to disable a person is directly proportional to the person's weight, how many bottles will disable someone weighing 210 pounds?

- _____ 5. What is the smallest possible value of n if $n > 1$ and $(1 + 2 + 3 + \dots + n)^2$ is a perfect fourth power?

- _____ 6. How many minutes are in $\frac{1}{10}$ of 2 weeks?

- _____ 7. Bob is currently 4 times as old as his sister. In 1 year, he will only be 3 times as old as his sister. In how many years will Bob only be 2 times as old as his sister?

- _____ 8. How many terms are in the arithmetic sequence 7, 11, 15, \dots , 127, 131?

- _____ 9. Jennifer pays \$4.02 for 3 apples and 2 oranges. Joseph pays \$6.46 for 1 apple and 6 oranges. What is the price of an apple in cents?
- _____ 10. Let $m = \frac{103!}{100!}$. Find the sum of the prime factors of m .
- _____ 11. At the restaurant Seyepop, fried chicken comes in packs of 2, 4, 6, 8, ... (any even integer) and 15. What is the maximum number of pieces of fried chicken that cannot be purchased using these packs at this Seyepop organization?
- _____ 12. Let $f(x) = 2x$ and let $g(x) = 3x - 3$. Find x such that $g(f(x)) = x$.
- _____ 13. A rectangular prism with positive integer edge lengths has a volume of 345. No edge is more than 20 times the length of another edge. What is the surface area of the prism?
- _____ 14. What is the sum of all values of x that satisfy $|x| = (2016 * 6102)^2$?
- _____ 15. The sum of 7 consecutive positive integers is 2016. Find the smallest of the 7 integers.
- _____ 16. Eric is running a 1600 meter race. Eric runs the first 100 meters in 10 seconds. For the rest of the race, Eric's next 100 meters will take him 1 second longer than the last 100 meters. How long will it take for Eric to finish the race?
- _____ 17. Let ABC be an equilateral triangle. We inscribe a regular n -gon in ABC such that the vertices of the n -gon each lie on a side of ABC . What is the maximum value of n ?

18. John is playing poker with a standard 52-card deck. How many ways are there to get a straight flush? A straight flush is 5 cards of consecutive rank like 2-3-4-5-6 or 10-J-Q-K-A that are also the same suit. The order of the cards doesn't matter. Note: A-2-3-4-5 and 10-J-Q-K-A are valid straights, but J-Q-K-A-2 is not.
19. If $v + w = 7$, $w + x = 20$, $x + y = 11$, and $y + z = 15$, compute $z - v$.
20. Let there be chords AB and CD in circle O such that AB and CD intersect at a point P inside of O . If $AP = 8$, $BP = 9$ and $CP = 6$, find the length DP .
21. Evaluate $\sqrt{90 + \sqrt{90 + \sqrt{90 + \dots}}}$
22. When a positive integer x is divided by 2, the remainder is 1. When x is divided by 3, the remainder is 2. When x is divided by 5, the remainder is 4. When x is divided by 7, the remainder is 6. When x is divided by 11, the remainder is 10. What is the smallest possible value of x ?
23. There are 6 balls in a bag. 2 are red, 2 are blue, and 2 are green. 4 balls are chosen at random, without replacement. What is the probability that there will be at least one of each color?
24. How many 3-digit positive integers have the property that the hundreds digit is strictly the greatest of all the digits?
25. Cire and Ymereg are playing a game where they alternate turns. The integers from 1 to 120 are placed in a hat and are to be drawn with replacement. Cire wins if he draws a multiple of 2, and Ymereg wins if he draws a multiple of 3. If neither player wins on their turn, the game continues. If Cire goes first, what is the probability that Ymereg wins?