

MBMT Team Round — Euclid Answers

1. How many positive divisors does 72 have?
Answer: 12
2. Raymond wants to travel in a car with 3 other (distinguishable) people. The car has 5 seats: a driver's seat, a passenger seat, and a row of 3 seats behind them. If Raymond's cello must be in a seat next to him, and he can't drive, but every other person can, how many ways can everyone sit in the car?
Answer: 24
3. Peter wants to make fruit punch. He has orange juice (100% orange juice), tropical mix (25% orange juice, 75% pineapple juice), and cherry juice (100% cherry juice). If he wants his final mix to have 50% orange juice, 10% cherry juice, and 40% pineapple juice, in what ratios should he mix the 3 juices? Please write your answer in the form (orange):(tropical):(cherry), where the three integers are relatively prime.
Answer: 11 : 16 : 3
4. Points A , B , C , and D are chosen on a circle such that $m\angle ACD = 85^\circ$, $m\angle ADC = 40^\circ$, and $m\angle BCD = 60^\circ$. What is $m\angle CBD$?
Answer: 55
5. a , b , and c are positive real numbers. If $abc = 6$ and $a + b = 2$, what is the minimum possible value of $a + b + c$?
Answer: 8
6. Circles A and B are drawn on a plane such that they intersect at two points. The centers of the two circles and the two intersection points lie on another circle, circle C . If the distance between the centers of circles A and B is 20 and the radius of circle A is 16, what is the radius of circle B ?
Answer: 12
7. Point P is inside rectangle $ABCD$. If $AP = 5$, $BP = 6$, and $CP = 7$, what is the length of DP ?
Answer: $\sqrt{38}$
8. For how many integers n is $n^2 + 4$ divisible by $n + 2$?
Answer: 8
9. How many of the perfect squares between 1 and 10000, inclusive, can be written as the sum of two triangular numbers? We define the n th triangular number to be $1 + 2 + 3 + \dots + n$, where n is a positive integer.
Answer: 99

10. A small sphere of radius 1 is sitting on the ground externally tangent to a larger sphere, also sitting on the ground. If the line connecting the spheres' centers makes a 60° angle with the ground, what is the radius of the larger sphere?
Answer: $7 + 4\sqrt{3}$
11. A classroom has 12 chairs in a row and 5 distinguishable students. The teacher wants to position the students in the seats in such a way that there is at least one empty chair between any two students. In how many ways can the teacher do this?
Answer: 6720
12. Let there be real numbers a and b such that $a/b^2 + b/a^2 = 72$ and $ab = 3$. Find the value of $a^2 + b^2$.
Answer: 75
13. Find the number of ordered pairs of positive integers (x, y) such that $\gcd(x, y) + \text{lcm}(x, y) = x + y + 8$.
Answer: 8
14. Evaluate $\sum_{i=1}^{\infty} \frac{i}{4^i} = \frac{1}{4} + \frac{2}{16} + \frac{3}{64} + \dots$
Answer: $\frac{4}{9}$
15. Xavier and Olivia are playing tic-tac-toe. Xavier goes first. How many ways can the game play out such that Olivia wins on her third move? The order of the moves matters.
Answer: 5328