MBMT Team Round — Euclid

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THIS TEST IN! You will turn in your answers on the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

1. How many positive divisors does 72 have?

2. Raymond wants to travel in a car with 3 other (distinguishable) people. The car has 5 seats: a driver's seat, a passenger seat, and a row of 3 seats behind them. If Raymond's cello must be in a seat next to him, and he can't drive, but every other person can, how many ways can everyone sit in the car?

3. Peter wants to make fruit punch. He has orange juice (100% orange juice), tropical mix (25% orange juice, 75% pineapple juice), and cherry juice (100% cherry juice). If he wants his final mix to have 50% orange juice, 10% cherry juice, and 40% pineapple juice, in what ratios should he mix the 3 juices? Please write your answer in the form (orange):(tropical):(cherry), where the three integers are relatively prime.

4. Points A, B, C, and D are chosen on a circle such that $m\angle ACD = 85°$, $m\angle ADC = 40°$, and $m\angle BCD = 60°$. What is $m\angle CBD$?

5. a, b, and c are positive real numbers. If $abc = 6$ and $a + b = 2$, what is the minimum possible value of $a + b + c$?

6. Circles A and B are drawn on a plane such that they intersect at two points. The centers of the two circles and the two intersection points lie on another circle, circle C. If the distance between the centers of circles A and B is 20 and the radius of circle A is 16, what is the radius of circle B?

7. Point P is inside rectangle $ABCD$. If $AP = 5$, $BP = 6$, and $CP = 7$, what is the length of DP?

8. For how many integers n is $n^2 + 4$ divisible by $n + 2$?

9. How many of the perfect squares between 1 and 10000, inclusive, can be written as the sum of two triangular numbers? We define the nth triangular number to be $1 + 2 + 3 + \ldots + n$, where n is a positive integer.

10. A small sphere of radius 1 is sitting on the ground externally tangent to a larger sphere, also sitting on the ground. If the line connecting the spheres' centers makes a 60° angle with the ground, what is the radius of the larger sphere?

11. A classroom has 12 chairs in a row and 5 distinguishable students. The teacher wants to position the students in the seats in such a way that there is at least one empty chair between any two students. In how many ways can the teacher do this?

12. Let there be real numbers a and b such that $a/b^2 + b/a^2 = 72$ and $ab = 3$. Find the value of $a^2 + b^2$.

13. Find the number of ordered pairs of positive integers (x, y) such that $\gcd(x, y)+\text{lcm}(x, y) = x + y + 8$.

14. Evaluate $\displaystyle\sum_{i=1}^{\infty} \frac{i}{4^i} = \frac{1}{4} + \frac{2}{16} + \frac{3}{64} + \ldots$

15. Xavier and Olivia are playing tic-tac-toe. Xavier goes first. How many ways can the game play out such that Olivia wins on her third move? The order of the moves matters.