Team # _____

MBMT Guts Round Set 1

______________ 1. Arnold is currently stationed at $(0,0)$. He wants to buy some milk at $(3,0)$, and also some cookies at $(0,4)$, and then return back home at $(0,0)$. If Arnold is very lazy and wants to minimize his walking, what is the length of the shortest path he can take?

______________ 2. Dilhan selects 1 shirt out of 3 choices, 1 pair of pants out of 4 choices, and 2 socks out of 6 differently-colored socks. How many outfits can Dilhan select? All socks can be worn on both feet, and outfits where the only difference is that the left sock and right sock are switched are considered the same.

______________ 3. What is the sum of the first 100 odd positive integers?

______________ 4. Find the sum of all the distinct prime factors of 1591.

______________ 5. Let set $S = \{1, 2, 3, 4, 5, 6\}$. From S, four numbers are selected, with replacement. These numbers are assembled to create a 4-digit number. How many such 4-digit numbers are multiples of 3?

Team # _____

MBMT Guts Round Set 2

______________ 6. What is the area of a triangle with vertices at $(0, 0)$, $(7, 2)$, and $(4, 4)$?

______________ 7. Call a number n "warm" if $n - 1$, n, and $n + 1$ are all composite. Call a number m "fuzzy" if m may be expressed as the sum of 3 consecutive positive integers. How many numbers less than or equal to 30 are warm and fuzzy?

______________ 8. Consider a square and hexagon of equal area. What is the square of the ratio of the side length of the hexagon to the side length of the square?

______________ 9. If $x^2 + y^2 = 361$, $xy = -40$, and $x - y$ is positive, what is $x - y$?

______________ 10. Each face of a cube is to be painted red, orange, yellow, green, blue, or violet, and each color must be used exactly once. Assuming rotations are indistinguishable, how many ways are there to paint the cube?

Team # _____

MBMT Guts Round Set 3

__________ 11. Let D be the midpoint of side BC of triangle ABC. Let P be any point on segment AD. If M is the maximum possible value of $\frac{[PAB]}{[PAC]}$ and m is the minimum possible value, what is $M - m$? (Note: $[PQR]$ denotes the area of triangle PQR.)

__________ 12. If the product of the positive divisors of the positive integer n is n^6, find the sum of the 3 smallest possible values of n.

__________ 13. Find the product of the magnitudes of the complex roots of the equation $(x-4)^4 + (x-2)^4 + 14 = 0$.

__________ 14. If $xy - 20x - 16y = 2016$ and x and y are both positive integers, what is the least possible value of $\max(x, y)$?

__________ 15. A peasant is trying to escape from Chyornarus, ruled by the Tsar and his mystical faith healer. The peasant starts at $(0, 0)$ on a 6×6 unit grid, the Tsar's palace is at $(3, 3)$, the healer is at $(2, 1)$, and the escape is at $(6, 6)$. If the peasant crosses the Tsar's palace or the mystical faith healer, he is executed and fails to escape. The peasant's path can only consist of moves upward and rightward along the gridlines. How many valid paths allow the peasant to escape?

Team # _____

MBMT Guts Round Set 4

_____ 16. Albert, Beatrice, Corey, and Dora are playing a card game with two decks of cards numbered 1-50 each. Albert, Beatrice, and Corey draw cards from the same deck without replacement, but Dora draws from the other deck. What is the probability that the value of Corey's card is the highest value or is tied for the highest value of all 4 drawn cards?

_____ 17. Suppose that s is the sum of all positive values of x that satisfy $2016\{x\} = x + [x]$. Find $\{s\}$. (Note: $[x]$ denotes the greatest integer less than or equal to x and $\{x\}$ denotes $x - [x]$.)

_____ 18. Let ABC be a triangle such that $AB = 41$, $BC = 52$, and $CA = 15$. Let H be the intersection of the B altitude and C altitude. Furthermore let P be a point on AH. Both P and H are reflected over BC to form P' and H'. If the area of triangle $P'H'C$ is 60, compute PH.

_____ 19. A random integer n is chosen between 1 and 30, inclusive. Then, a random positive divisor of n, k, is chosen. What is the probability that $k^2 > n$?

_____ 20. What are the last two digits of the value 3^{361}?

Team # _____

MBMT Guts Round Set 5

_____ 21. Let $f(n)$ denote the number of ways a $3 \times n$ board can be completely tiled with 1×3 and 1×4 tiles, without overlap or any tiles hanging over the edge. The tiles may be rotated. Find $\sum_{i=0}^{9} f(i) = f(0) + f(1) + \ldots + f(8) + f(9)$. By convention, $f(0) = 1$.

_____ 22. Find the sum of all 5-digit perfect squares whose digits are all distinct and come from the set $\{0, 2, 3, 5, 7, 8\}$.

_____ 23. Mary is flipping a fair coin. On average, how many flips would it take for Mary to get 4 heads and 2 tails?

_____ 24. A cylinder is formed by taking the unit circle on the xy-plane and extruding it to positive infinity. A plane with equation $z = 1 - x$ truncates the cylinder. As a result, there are three surfaces: a surface along the lateral side of the cylinder, an ellipse formed by the intersection of the plane and the cylinder, and the unit circle. What is the total surface area of the ellipse formed and the lateral surface? (The area of an ellipse with semi-major axis a and semi-minor axis b is πab.)

_____ 25. Let the Blair numbers be defined as follows: $B_0 = 5$, $B_1 = 1$, and $B_n = B_{n-1} + B_{n-2}$ for all $n \geq 2$. Evaluate $\sum_{i=0}^{\infty} \frac{B_i}{51^i} = B_0 + \frac{B_1}{51} + \frac{B_2}{51^2} + \frac{B_3}{51^3} + \ldots$

Team # _____

MBMT Guts Round Estimation

_____________ 26. Choose an integer between 1 and 10, inclusive. Your score will be the number you choose divided by the number of teams that chose your number.

_____________ 27. 2016 blind people each bring a hat to a party and leave their hat in a pile at the front door. As each partier leaves, they take a random hat from the ones remaining in a pile. Estimate the probability that at least 1 person gets their own hat back.

_____________ 28. Estimate how many lattice points lie within the graph of $|x^3| + |y^3| < 2016$.

_____________ 29. Consider all ordered pairs of integers (x, y) with $1 \leq x, y \leq 2016$. Estimate how many such ordered pairs are relatively prime.

_____________ 30. Estimate how many times the letter "e" appears among all Guts Round questions.