

## MBMT Algebra Round — Lobachevsky Answers

\_\_\_\_\_ 1. What is the largest integer  $n$  for which  $n^{24} < 20^{16}$ ?

Answer: 7

\_\_\_\_\_ 2. The sequence  $E_n$  has the property that  $E_1 = 20$  and  $E_n = E_{n-2} + E_{n-1}$  for all  $n \geq 3$ . If  $E_5 = 16$ , what is  $E_7$ ?

Answer: 36

\_\_\_\_\_ 3. Let  $f(x) = x^3 + 1729x^2 + 1728x + 1727$ . Find the sum of the roots of  $f(x + 2)$ .

Answer:  $-1735$

\_\_\_\_\_ 4. Danny and Jason each choose a positive integer. They notice that Danny's integer, Jason's integer, and their product minus 4 times Danny's integer form an arithmetic sequence, in that order. Let  $a$  be Danny's integer and  $b$  be Jason's integer. What are all possible ordered pairs  $(a, b)$ ?

Answer:  $(3, 9), (4, 6), (5, 5), (8, 4)$

\_\_\_\_\_ 5. Compute  $50 * 50 + 51 * 49 + 52 * 48 + \dots + 99 * 1 + 100 * 0$ .

Answer: 84575

\_\_\_\_\_ 6.  $f(x)$  has the property that  $5f(x) - 3f\left(\frac{1}{x}\right) = x^3$  for all nonzero  $x$ . Find  $f(\sqrt[3]{6})$ .

Answer:  $\frac{61}{32}$

\_\_\_\_\_ 7. Evaluate  $\sum_{i=0}^{\infty} \frac{2}{(n+1)(n+5)} = \frac{2}{1*5} + \frac{2}{2*6} + \frac{2}{3*7} + \dots$

Answer:  $\frac{25}{24}$

\_\_\_\_\_ 8. Let  $f(x)$  be a function such that  $f(x)f(y) - f(xy) = xy$  for all real  $x$  and  $y$ . Let  $M$  and  $m$  be the maximum possible value and minimum possible value, respectively, of  $f(2016)$ . Find  $M - m$ .

Answer:  $2016\sqrt{5}$