

## MBMT Geometry Round — Lobachevsky Answers

1. Let  $ABCD$  be a square with side length 30. A circle centered at the center of  $ABCD$  with diameter 34 is drawn. Let  $E$  and  $F$  be the points at which the circle intersects side  $AB$ . What is  $EF$ ?  
Answer: 16
2. What is the area of the quadrilateral bounded by  $|2x| + |3y| = 6$ ?  
Answer: 12
3. Right triangle  $ABC$  has hypotenuse  $AB$ . Altitude  $CD$  divides  $AB$  into segments  $AD$  and  $DB$ , with  $AD = 20$  and  $DB = 16$ . What is the area of triangle  $ABC$ ?  
Answer:  $144\sqrt{5}$
4. Circle  $O$  has chord  $AB$ . Extend  $AB$  past  $B$  to a point  $C$ . A ray from  $C$  is drawn, and this ray intersects circle  $O$ . Let point  $D$  be the point of intersection of the ray and the circle that is closest to point  $C$ . Given  $AB = 20$ ,  $BC = 16$ , and  $OA = \frac{201}{6}$ , find the longest possible length of  $CD$ .  
Answer: 24
5. Consider a circular cone with vertex  $A$ . The cone's height is 4 and the radius of its base is 3. Inscribe a sphere inside the cone. Find the ratio of the volume of the cone to the volume of the sphere.  
Answer:  $\frac{8}{3}$
6. A disk of radius  $\frac{1}{2}$  is randomly placed on the coordinate plane. What is the probability that it contains a lattice point (point with integer coordinates)?  
Answer:  $\frac{\pi}{4}$
7. Let  $ABC$  be an equilateral triangle of side length 2. Let  $D$  be the midpoint of  $BC$ , and let  $P$  be a variable point on  $AC$ . By moving  $P$  along  $AC$ , what is the minimum perimeter of triangle  $BDP$ ?  
Answer:  $1 + \sqrt{7}$
8. Let  $ABCD$  be a rectangle with  $AB = 8$  and  $BC = 9$ . Let  $DEFG$  be a rhombus, where  $G$  is on line  $BC$  and  $A$  is on line  $EF$ . If  $m\angle EFG = 30^\circ$ , what is  $DE$ ?  
Answer: 12