MBMT Sprint Round — Lobachevsky Answers

___________ 1. What is the remainder when the positive integers from 1 to 7 are added together and then divided by 6?
Answer: 4

___________ 2. What is the units digit of the product of the first 10 primes?
Answer: 0

___________ 3. What is the smallest possible value of n if $n > 1$ and $(1+2+3+\cdots+n)^2$ is a perfect fourth power?
Answer: 8

___________ 4. How many terms are in the arithmetic sequence $7, 11, 15, \ldots, 127, 131$?
Answer: 32

___________ 5. Let $m = \frac{103!}{100!}$. Find the sum of the prime factors of m.
Answer: 226

___________ 6. Let $f(x) = 2x$ and let $g(x) = 3x - 3$. Find x such that $g(f(x)) = x$.
Answer: $\frac{3}{5}$

___________ 7. A sphere is intersected with a regular tetrahedron. What is the maximum number of intersection points which lie on the edges of the regular tetrahedron?
Answer: 12

___________ 8. At the restaurant Seyepop, there is one item on the menu fried chicken. Fried chicken comes in packs of $2, 4, 6, 8, \ldots$ (any even integer) and 15. What is the maximum number of pieces of fried chicken that cannot be purchased using these packs at this Seyepop organization?
Answer: 13

___________ 9. How many ordered pairs of integers (x, y) satisfy the equation $4x^2 - y^2 + 1 = 0$?
Answer: 2

__________ 10. How many of the first 2016 triangular numbers are odd? We define the nth triangular number to be $1 + 2 + 3 + \ldots + n$, where n is a positive integer.
Answer: 1008

__________ 11. Find the remainder when 2016^{2016} is divided by 31.
Answer: 1

__________ 12. Let there be chords AB and CD in circle O such that AB and CD intersect at a point P inside of O. If $AP = 8$, $BP = 9$ and $CP = 6$, find the value of DP.
Answer: 12

__________ 13. Evaluate $\sqrt{90 + \sqrt{90 + \sqrt{90 + \ldots}}}$.
Answer: 10

__________ 14. Circle O has radius 1. Points A and B on circle O are chosen such that $m\angle AOB = 120^\circ$. What is the area of the smaller region bound by the circle and segment AB?
Answer: $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

__________ 15. Alice and Bob are playing a game. Alice rolls 4 6-sided dice and finds the sum, while Bob rolls a single n-sided die. If they get the same number, they both roll again until someone's result is higher. If both people have an equal chance of winning, what is n?
Answer: 27

__________ 16. When a positive integer x is divided by 2, the remainder is 1. When x is divided by 3, the remainder is 2. When x is divided by 5, the remainder is 4. When x is divided by 7, the remainder is 6. When x is divided by 11, the remainder is 10. What is the smallest possible value of x?
Answer: 2309

__________ 17. There are 6 balls in a bag. 2 are red, 2 are blue, and 2 are green. 4 balls are chosen at random, without replacement. What is the probability that there will be at least one of each color?
Answer: $\frac{4}{5}$

__________ 18. Mr. Lodal is on a field trip star gazing with his earth science class. Every star is a regular star with $2n$ vertices, which is created by taking the sides of a regular n-gon ($n \geq 5$) and extending the sides until two originally non-adjacent sides meet. Mr. Lodal determines that in a certain type of star, the non-reflex angles are equal to 135 degrees. Determine the number of vertices in this type of star.
Answer: 32

__________ 19. Evaluate $162^3 - 3 * 162^2 * 160 + 3 * 160^2 * 162 - 160^3$.
Answer: 8

__________ 20. How many 3-digit positive integers have the property that the hundreds digit is strictly the greatest of all the digits?
Answer: 285

__________ 21. What is the height of a regular tetrahedron with side length 2?
Answer: $\frac{2\sqrt{6}}{3}$

__________ 22. Let m denote the minimum number of distinct lines necessary to cut the plane into 2016 sections and n denote the maximum number of distinct lines that can cut the plane into exactly 2016 sections. Compute $n - m$.
Answer: 1952

__________ 23. Cire and Ymerej are playing a game where they alternate turns. The integers from 1 to 120 are placed in a hat and are to be drawn with replacement. Cire wins if he draws a multiple of 2, and Ymerej wins if he draws a multiple of 3. If neither player wins on their turn, the game continues. If Cire goes first, what is the probability that Ymerej wins?
Answer: $\frac{1}{4}$

__________ 24. If $a + b + c = 1$ and $a^3 + b^3 + c^3 = \frac{1}{6}$, find the value of $(a + b)(a + c)(b + c)$.
Answer: $\frac{5}{18}$

__________ 25. How many ways can 3 characters be chosen from "$AAABBBCCCDDEEFFGHI$" if the characters representing each letter are indistinguishable, each character may only be used once, and order matters? Two such ways are "AAB" and "BAA".
Answer: 651