

MBMT Team Round — Lobachevsky Answers

1. What is the sum of the positive divisors of 100?
Answer: 217
2. Raymond wants to travel in a car with 3 other (distinguishable) people. The car has 5 seats: a driver's seat, a passenger seat, and a row of 3 seats behind them. If Raymond's cello must be in a seat next to him, and he can't drive, but every other person can, how many ways can everyone sit in the car?
Answer: 24
3. Peter wants to make fruit punch. He has orange juice (100% orange juice), tropical mix (25% orange juice, 75% pineapple juice), and cherry juice (100% cherry juice). If he wants his final mix to have 50% orange juice, 10% cherry juice, and 40% pineapple juice, in what ratios should he mix the 3 juices? Please write your answer in the form (orange):(tropical):(cherry), where the three integers are relatively prime.
Answer: 11 : 16 : 3
4. Points A , B , C , and D are chosen on a circle such that $m\angle ACD = 85^\circ$, $m\angle ADC = 40^\circ$, and $m\angle BCD = 60^\circ$. What is $m\angle CBD$?
Answer: 55
5. Circles A and B are drawn on a plane such that they intersect at two points. The centers of the two circles and the two intersection points lie on another circle, circle C . If the distance between the centers of circles A and B is 20 and the radius of circle A is 16, what is the radius of circle B ?
Answer: 12
6. For how many integers n is $n^2 + 4$ divisible by $n + 2$?
Answer: 8
7. A small sphere of radius 1 is sitting on the ground externally tangent to a larger sphere, also sitting on the ground. If the line connecting the spheres' centers makes a 60° angle with the ground, what is the radius of the larger sphere?
Answer: $7 + 4\sqrt{3}$
8. A classroom has 12 chairs in a row and 5 distinguishable students. The teacher wants to position the students in the seats in such a way that there is at least one empty chair between any two students. In how many ways can the teacher do this?
Answer: 6720
9. Let there be real numbers a and b such that $a/b^2 + b/a^2 = 72$ and $ab = 3$. Find the value of $a^2 + b^2$.
Answer: 75

10. Find the number of ordered pairs of positive integers (x, y) such that $\gcd(x, y) + \text{lcm}(x, y) = x + y + 8$.

Answer: 8

11. Evaluate $\sum_{i=1}^{\infty} \frac{i}{4^i} = \frac{1}{4} + \frac{2}{16} + \frac{3}{64} + \dots$

Answer: $\frac{4}{9}$

12. Xavier and Olivia are playing tic-tac-toe. Xavier goes first. How many ways can the game play out such that Olivia wins on her third move? The order of the moves matters.

Answer: 5328

13. Let $ABCD$ be a convex quadrilateral with $AC = 20$. Furthermore, let $M, N, P,$ and Q be the midpoints of $DA, AB, BC,$ and $CD,$ respectively. Let X be the intersection of the diagonals of quadrilateral $MNPQ$. Given that $NX = 12$ and $XP = 10$, compute the area of $ABCD$.

Answer: 384

14. Evaluate $(\sqrt{3} + \sqrt{5})^6$ to the nearest integer.

Answer: 3904

15. In Hatland, each citizen wears either a green hat or a blue hat. Furthermore, each citizen belongs to exactly one neighborhood. On average, a green-hatted citizen has 65% of his neighbors wearing green hats, and a blue-hatted citizen has 80% of his neighbors wearing blue hats. Each neighborhood has a different number of total citizens. What is the ratio of green-hatted to blue-hatted citizens in Hatland? (A citizen is his own neighbor.)

Answer: $\frac{4}{7}$